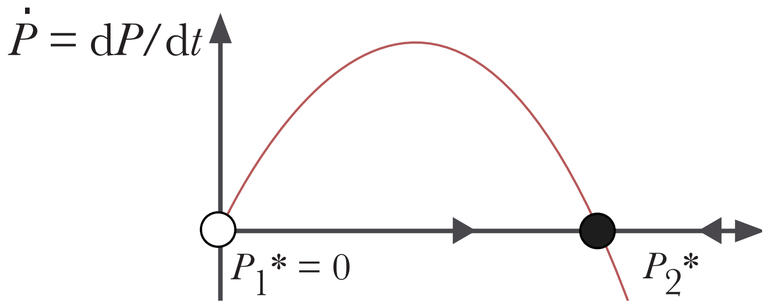
Predicting Environmental System Collapse.

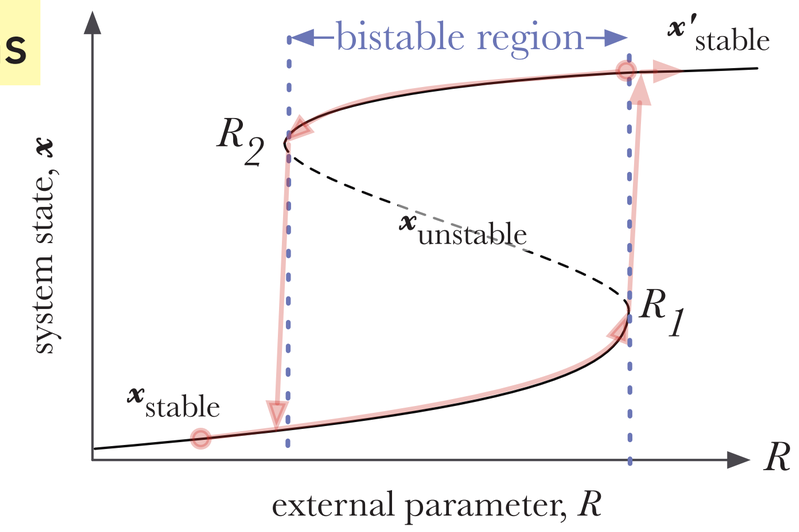
Abstract: An enormous array of potential catastrophes facing humanity in the 21st century rely on our ability to accurately forecast the likelihood and impact of behavioural changes in complex systems, such as the global climate system. A growing number of simple, generic statistical measures have been proposed to give early warning for incipient complex system collapse, which are critically evaluated here for two critical environmental case studies: Atlantic Transhaline Circulation (THC) collapse and lake eutrophication.

Dynamical Systems and Bifurcations



Dynamical systems formalises a system whose state (as a d-dimensional vector) changes deterministically over time as an ODE (Ordinary Differential Equation).

The above demonstrates a simple model for population growth (P). There is an unstable fixed point at P_1^* (we diverge unless $P = P_1^*$ exactly). There is a stable fixed point at P_2^* (perturbations away from P_2^* decay back to P_2^*).



Fold Bifurcation Model

A **bifurcation** is some change the properties of a dynamical system in response to external forcing parameter, R. The region $R_2 < R < R_1$ has two stable points. We see a sudden, discontinuous jump between stable states in response to a small increase in R. A small reversal of forcing does not return the system to previous state (**hysteresis**).

Atlantic Transhaline Circulation

Deepwater currents driven by differences in density between water masses (from temperature/salinity differences). Arctic/Greenland ice sheet meltwater affects salinity & local air temperature in formation regions.

THC plays a significant role in moderating Western European climate. There isevidence of fold bifurcational behaviour from state-o-the-art climate models (GENIE-2)[3].

Lake Eutrophication

Sudden algal blooming in lakes driven by slow increase in phosphates from agricultural runoff. This significantly affects biodiversity and aesthetic value of lake systems. Used as a classic example of bistable behaviour from models[6] and empirical studies[1].

Early Warning Signals (EWS)

Critical Slowing Down

Saddle node bifurcation $x = x^2 + r$ (bifurcation at $r = 0$) has stable fixed point a $x_1^* = -\sqrt{r}$ with basin of attraction $B = 2\sqrt{r}$. Consider a perturbation ε:

$$\begin{aligned}\frac{d}{dt}[x_1^* + \varepsilon] &= f(x_1^* + \varepsilon) \simeq f(x_1^*) + \left.\frac{\partial f}{\partial x}\right|_{x=x_1^*} \varepsilon \\ &= f(x_1^*) - 2\varepsilon\sqrt{r} \\ &= f(x_1^*) + \lambda\varepsilon, \text{ such that } \lambda = -B.\end{aligned}$$

The rate of decay, λ, decreases linearly with the size of the basin (B) as we approach the bifurcation point $r = 0$. We can infer critical slowing from natural, fast stochastic perturbations (e.g. short-term weather dynamics) [2].

Discrete Time Series Statistics

Slowing can be inferred from statistics of fixed-length windows of the the dependent variable time series: $Y_1, \ldots Y_n$.

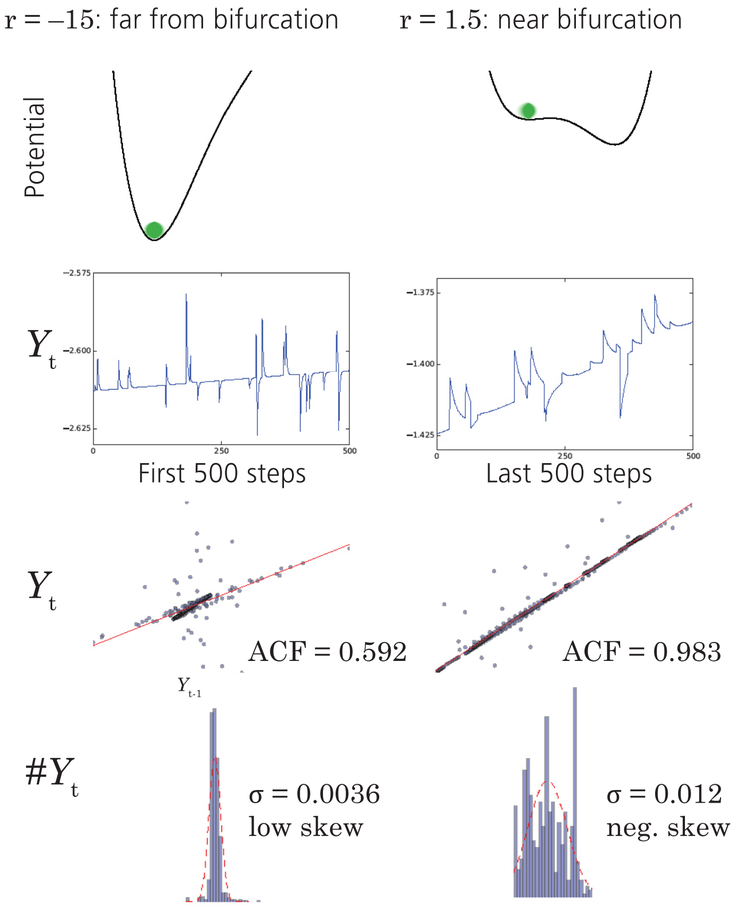
Lag-1 Autocorrelation (ACF)
Each time step is more "like" the previous – increase in correlation between Y_t and Y_{t+1}.

Detrended Fluctuation Analysis (DFA) [7]
Picks up long-range power law fractal self-similarity in integrated time series.

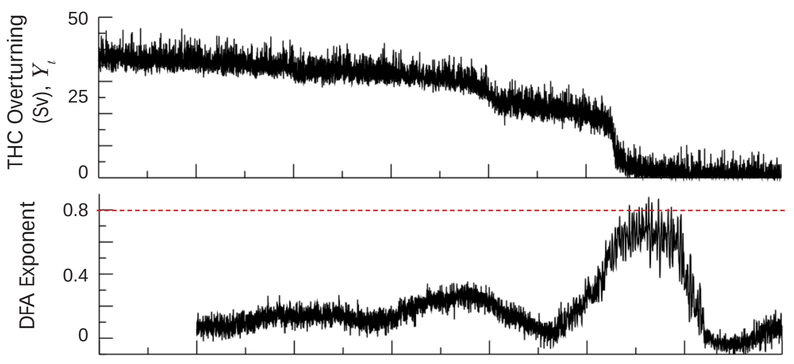
Variance
As the attracting well becomes wider, we expect to see more varied values.

Other Signals & Phenomena
- **asymmetry** – system oscillating between two alternative stable basins (flickering) and shallower gradient by saddle. Increased variance and skewness
- **"reddening"** of the spectral properties of the time series.

Synthetic experiments with $r + 3x - x^3 = 0$, with $r \in [-15, 1.5]$, and noise $\eta \sim \mathcal{N}(0, 0.01)$ with probability $1/25$ [8]:



Supporting Evidence



- simulations of THC collapse on GENIE-2 have shown increased DFA power law exponent (above) [3]
- analysis shows increased ACF [2] and DFA [7] in paleo records preceding many major warming events
- more success predicting lake eutrophication in models through increased variance/skewness [4].

Limitations

- little a priori reason for many of the assumptions [1]:
  - approaching a saddle-node bifurcation – other non-catastrophic bifurcations also exhibit slowing
  - time scales for parameter drift ≫ decay ≫ noise
  - slow monotonic drift of forcing parameter – the transition could be induced by stochastic noise or nonlinear forcing.
- much of the evidence for early warning signals falls for the prosecutor's fallacy [5]:
  - evidence uses series' with known transition (ignores prior)
  - High $\mathrm{P}(\mathrm{EWS} \mid \mathrm{Transition}) \not\Rightarrow$ High $\mathrm{P}(\mathrm{Transition} \mid \mathrm{EWS})$
  - many EWS (esp. ACF) likely to increase in normal conditions
  - ROC analysis on samples from explicit hypothesis testing to quantify classification ability.
- explicit process-based modelling has much closer link to underlying theory – greater insight and usually greater prediction certainty and precision.

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References:
[1] Scheffer, Marten, et al. "Early-warning signals for critical transitions." *Nature* 461.7260 (2009): 53-59.
[2] Thompson J. Michael T., and Jan Sieber. "Predicting climate tipping as a noisy bifurcation: a review" *International Journal of Bifurcation and Chaos* 21.02 (2011): 399-422.
[3] Lenton, Timothy M., et al. "Using GENIE to study a tipping point in the climate system." *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 367.1890 (2009): 871-884.
[4] Dakos, Vasilis, et al. "Slowing down as an early warning signal for abrupt climate change." *Proceedings of the National Academy of Sciences* 105.38 (2008): 14308-14312.
[5] Boettiger, Carl, and Alan Hastings. "Quantifying limits to detection of early warning for critical transitions." *Journal of the Royal Society Interface* 9.75 (2012): 2527-2539.
[6] Scheffer, Marten, and Egbert H. van Nes. "Shallow lakes theory revisited: various alternative regumes driven by climate, nutrients, depth and lake size." *Hydrobiologia* 584.1 (2007): 455-466.
[7] Livina, Valerie N. and Timothy M. Lenton. "A modified method for detecting incipient bifurcations in a dynamical system." *Geophysical Research Letters* 34.3 (2007).
[8] Whitehall, Sam G. "Simulation of Critical Slowing from Stochastic Noise in a Fold Bifurcation." N.p., 13 May 2014. Web. 13 May 2014. <http://nbviewer.ipython.org/github/samwhitehall/critical-slowing-down/blob/master/Critical%20Slowing%20Down.ipynb>